Powder Technology 342 (2019) 594–606

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

Some observations on the influence of particle size and size distribution on stratification in pneumatic jigs

Weslei M. Ambrós [a,*], Carlos H. Sampaio [a], Bogdan G. Cazacliu [b], Paulo N. Conceição [a], Glaydson S. dos Reis [a,b]

[a] Mineral Processing Laboratory, Federal University of Rio Grande do Sul, 9500 Bento Gonçalves Avenue, Zip Code: 91501-970 Porto Alegre, Brazil
[b] LUNAM, IFSTTAR, Aggregates and Materials Processing Laboratory, Route de Bouaye – CS4, 44344 Bouguenais Cedex, Nantes, France

ARTICLE INFO

Article history:
Received 6 June 2018
Received in revised form 17 September 2018
Accepted 14 October 2018
Available online 16 October 2018

Keywords:
Pneumatic jig
Particle size
Size distribution
Stratification

ABSTRACT

Particle size variation plays a key role in jigging performance, and despite extensive research in the area, very little attention has been given in the case of pneumatic jigging. The aim of this study was to look into particle stratification in a pilot-scale pneumatic jig when varying the particle size and the range of the particle size distribution in ternary mixtures of aggregates. Jigging tests were especially designed to reduce contamination of jig products and a stratification index was elaborated to evaluate stratification efficiency. Experimental results provided compelling evidences that widening the particle size distribution of the system or using beds composed of particles of smaller sizes can enhance stratification by density. Similarly, smaller particles showed a remarkable tendency to concentrate in the upper zones of the stratified bed, whereas larger particles tended to concentrate more in lower zones. The obtained results suggest that particular operating features of pneumatic jigging together with differential packing effects should play a decisive role in the stratification extent of beds formed by particles of different sizes. Experimental results are of practical importance since, among other benefits, they point to the possibility to increase pneumatic jigging performance in some cases by using wider size distributions of the feed, thus reducing the need of prior stages of narrow size classification.

1. Introduction

Pneumatic jigging has attracted significant interest over recent years, especially due to the inherent advantage of not using process water. Though it is strongly associated with coal beneficiation [1–3], in recent years, pneumatic jigging has been extensively studied in urban mining applications, such as recycling of construction and demolition wastes [4–6], recovery of metals from electronic wastes [7] and separation of copper wires from rubber insulators [8]. However, its separation performance remains significantly lower than that of conventional hydraulic jigs, which eventually impose severe limits on its application [9]. Nonetheless, the great majority of studies focused on conventional hydraulic jigs and little attention has been paid to the effect of bed properties (particle density, size and shape) on stratification under dry conditions.

1.1. Overview of jigging process

Jigging has been known for centuries as an ore concentration process [10]. Hydraulic jigs of different types are widely used in several applications, covering every density range from less than 1.3 g/cm^3 for some coals [11] up to 19 g/cm^3 for gold [12], whereas pneumatic jigs is still more limited to coal and solid waste processing [1,3]. The jigging process consists in the repeated expansion and contraction of a non-homogeneous particle bed, producing stratification based upon specific gravity [9,13,14]. An illustration of the jigging action is shown in Fig. 1. During the pulsation stroke, the bed initially at rest on a rigid screen is lifted as a whole when the fluid flow exceeds the final falling velocity of the bed forming a porous rigid mass. As the fluid velocity is lowered and the upward motion of the bed slows towards its maximum displacement, a loosening layer starts as particles began to settle out from the bottom upwards. This loosening wave moves upwards through the bed until it reaches its maximum void fraction, so that particles fall under conditions analogous to hindered settling [14]. The heavier, larger particles tend to reach the jigging screen first than the lighter, smaller particles due to their higher specific gravities [15]. If the fluid medium is water, then the downward motion of particles is also influenced by the descending motion of the fluid (suction stroke). Otherwise, if the medium is air, so the suction stroke is virtually absent. The cumulative effect of several jigging cycles gives rise to a stratified bed where heavier, larger particles tends to concentrate in the lower layers whereas the lighter, finer particles tends to accumulate in the upper layers. Since density (not size) is a characteristic property of

* Corresponding author.
E-mail addresses: weslei.ambros@ufrgs.br (W.M. Ambrós), sampaio@ufrgs.br (C.H. Sampaio), bogdan.cazacliu@ifsttar.fr (B.G. Cazacliu), paulo.conceicao@ufrgs.br (P.N. Conceição), glaydson.simoes@ufrgs.br (G.S. dos Reis).

https://doi.org/10.1016/j.powtec.2018.10.029
0032-5910/

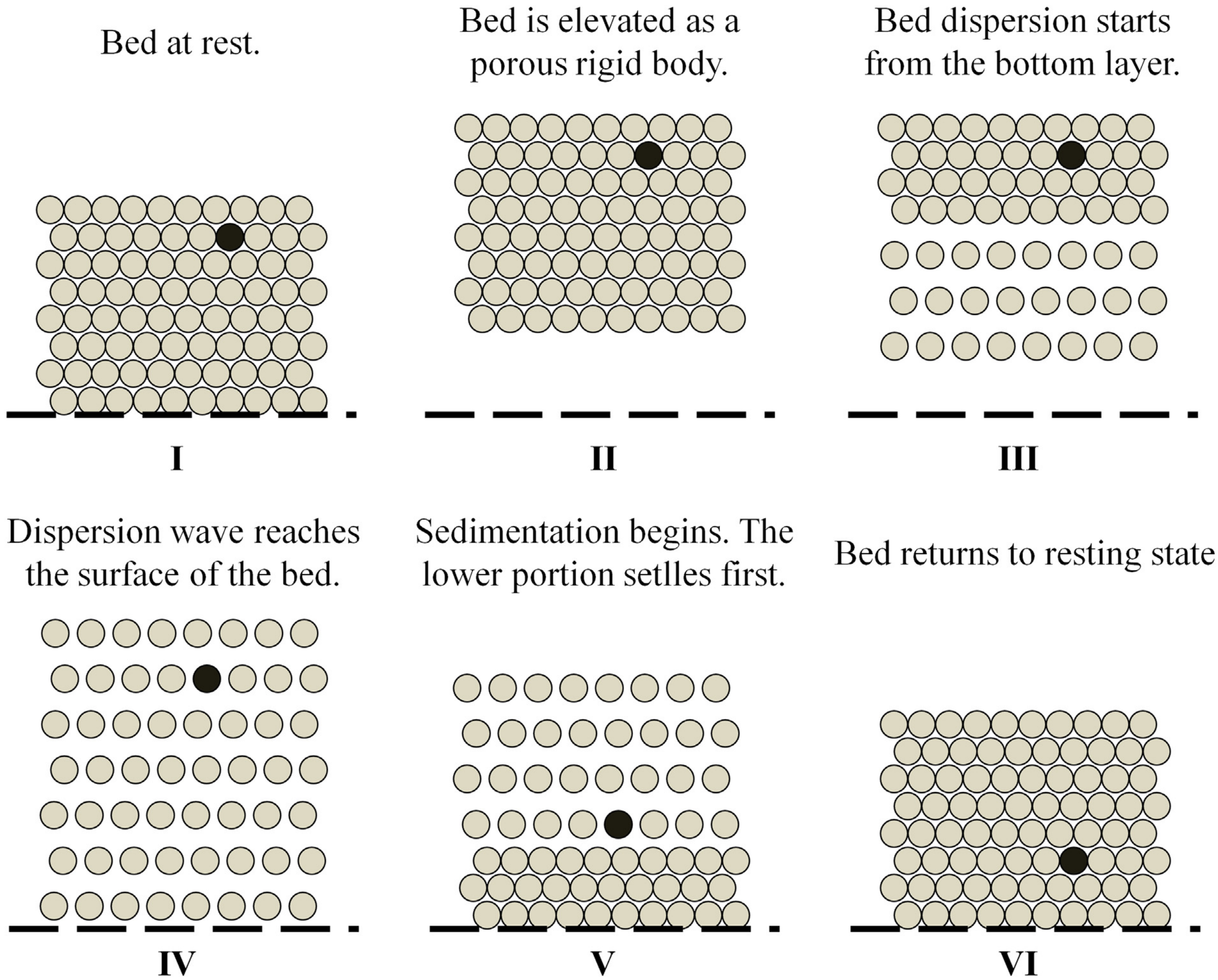

Fig. 1. Movement of the jigging bed subjected to vertical harmonic pulsation of the medium. The change of position of a heavy particle in the bed is represented by the black particle. Based on the description found in Kirchberg & Hentzschel [14] and Sampaio & Tavares [9].

different materials, jigging process is always targeted to maximize separation by density, instead of separation by size.

Provided that there is vertical motion of the bed, the mentioned mechanism is expected to occur independent of pulsation amplitude and frequency [14]. However, depending on these and on bed thickness, the loosening wave may or may not propagate through the whole bed. Also, the resulting stratification profile will depend on the properties of the bed and the existing differences in particle density, size and shape. Two main theories are pointed out in the description of particle stratification in jigs. One of them considers that jigging process can be described from the balance of forces acting in each individual particle of the system. Firstly introduced by Gaudin [16], this approach is currently used to develop coupled models based on the discrete element method (DEM) and computational fluid dynamics (CFD) [17,18]. The other model considers the mixed state of the jigging bed as a thermodynamic system which strives to achieve the state of lowest energy when pulsed, being that energy reduction the driving force behind the stratification [19]. More specifically, it assumes that the energy supplied by the fluid releases the potential energy stored in the particle bed, which is partially converted into lifting work done by the heavier particles to displace upward the lighter particles (other part is wasted as heat or friction). An extension of the potential energy theory was developed by Tavares and King [20] to include dispersion forces which prevent the bed to achieve the ideal stratification. Both models provide a plausible description (if not quantitatively, at least qualitatively) of the majority of phenomena involved in the stratification process (e.g., the occurrence of percolation trickling, the expected influence of particle shape, etc). However, some peculiar phenomena reported in recent studies, such as the occurrence of horizontal stratification patterns after jigging [21], suggests that the mechanisms involved in particle stratification in jigs still remains not fully understood.

1.2. Pneumatic jigging

Jigs can be categorized into several types depending on their design and operational features. The pulsating mechanism, method of products discharge, condition of the jig screen and the medium fluid are typical criteria used to distinguish among the different types of jigs. A brief discussion on the influence of medium fluid is presented here, but a more detailed review of the several jig types as well as their applications can be found in Sampaio & Tavares [9], Lyman [10] and Wills & Finch [13].

Besides water, jigs can operate with air as the medium. These are known as pneumatic jigs, air jigs or dry jigs (for the sake of clarity, we have chosen the designation *pneumatic jigs* in order to distinguish it from air-pulsed hydraulic-type jigs). Their main advantage in comparison to conventional hydraulic jigs is the elimination of water treatment and disposal circuits, and is particularly useful in locations where water is expensive or scarce [1]. In pneumatic jigs, the jig bed is pulsed by an upward air flow produced by a fan. Despite this different pulsating mechanism, the bed motion and the mechanisms of stratification in pneumatic jigs are expected to follow the same basic principles described for hydraulic jigs [9]. However, in contrast to water, the density of air is negligible in comparison to the particles of the bed. In order to compare the effect of varying the medium density on the stratification process, it is convenient to turn to the *concentration criterion* [22], an index largely used to estimate the ease at which materials can be separated by density, given by:

$$CC = \frac{\rho_H - \rho_f}{\rho_L - \rho_f} \tag{1}$$

where ρ_f is the density of the fluid, in g/cm^3; ρ_H is the density of the heavy material, in g/cm^3; and ρ_L is the density of the light material, in g/cm^3.

Larger values of CC indicate an easy separation by gravity methods and vice versa. Also, the larger the value of CC, the wider the particle size range in which separation by density can be efficiently carried out. Eq. (1) still indicates that the value of CC increases as the density of the fluid gets closer to the density of the light constituent. Consequently, separation by density is easier accomplished in wet jigging than in air jigging. Part of the complication is that the low density of air needs to be compensated with the use of high upward velocities, which, together with the difficulty to maintain a uniform distribution of the air over the whole cross-section of the bed, can produce excessive turbulence and thereby increase remixing effects. In addition, the use of air as medium make more difficult to separate very fine particles since the relatively small drag force acting on them need to compete with frictional and surface forces, which play a greater role in fine particles [23]. The practical consequence is that pneumatic jigs are used to separate only relatively coarse material, normally not smaller than 2 mm [5], and usually operate with close-sized feed. By contrast, conventional water jigs operates with a lower size limit of 0.5 mm, reaching up to 0.1 mm in the case of some metallic ores [9].

1.3. Influence of particle size on bed stratification

The role of particle size on density separation has been extensively investigated for the case of hydraulic jigs. A summary of these experimental studies are shown in Table 1. In spite of differences in experimental conditions, a definite trend has been reported in most cases: separation efficiency increased with increasing particle size of the jig bed. Also, size segregation can harm separation by density when jigging materials containing a wide range of particle sizes, which can be avoided by reducing the size range of the feed.

Segregation by size, which involves geometric interactions among different species/individual particles, can display complex patterns due to its non-monotonic behavior, since the segregation order can be varied and even reversed due to changes of size ratios between small and large particles [24]. In a recent study, Woollacott and Silwamba [25] examined this phenomenon in detail using binary mixtures of glass beads in a water batch jig. The results revealed the existence of at least four different patterns of size segregation according to the size ratio of the particles in the system.

In the case of dry jigging, Weinstein and Snoby [1] compared historical performance data of pneumatic jigs used in coal beneficiation against results from pneumatic jigs installed in modern coal plants, observing that in both cases jigs showed a lower performance when the feed particle size was decreased. However, to the knowledge of the authors, there is a lack of studies focusing exclusively on the effect of particle size on stratification in the framework of pneumatic jigging. One could think that some insights into this issue could be presumed a priori from data obtained from other gravitational air separators, such as counterflow classifiers, fluidized bed classifiers and pulsing air classifiers [26,27]. However, most of them work using free-fall separation system and continuous air stream, besides being extensively used for size separation only. On the other hand, like jigs, pulsing air classifiers uses a pulsed air flow to minimize the influence of particle size and thus achieve a density dominant separation (source). Nonetheless, the separation zones in air pulsed classifiers are designed to originate two distinct streams (overflow and underflow products), so providing a high dispersion of particles, which consequently experience much fewer interactions among each other in comparison to the dense, packed bed found in jigs.

It is worth to note that few studies have explored the possible existing links between size segregation in jigs and the mechanisms of size segregation in similar granular systems. For example, Schröter et al. [28] reported that at least seven distinct mechanisms can be considered when describing size segregation in beds subjected to vertical vibration (namely: void filling, static compressive force, convection, condensation, thermal diffusion, and two types of non-equipartition). Similarly, Metzger et al. [29] and Jain et al. [30] recently showed that size segregation in binary size mixtures could be reduced by the introduction of intermediate sized species. The obvious synergy between these studies and the phenomena involved in jigging indicate the existence of a vast field of study regarding dry jigging technology.

1.4. Paper objectives

The present study focuses on examining the effect of varying particle size and the range of the particle size distribution on stratification by density in a pilot-scale pneumatic jig. For this purpose, a procedure aiming to minimize experimental errors has been developed and a stratification index has been proposed as a useful way to quantify

Table 1
Summary of the selected literature with their main observations on the effect of particle size on density separation in jigs.

Reference	Jigging device	System	Efficiency assessment	Main observations
Olajide and Cho [31]	Laboratory-scale Baum jig	Coal samples in the size ranges of 19.1–12.7 mm, 12.7–6.4 mm and 6.4–3 mm.	Partition curves	Separation efficiency increased with increasing particle size;
Rong and Lyman [32]	Pilot-scale Baum jig	Density tracers (1.3 to 2 g/cm^3) in the size ranges of 35–16 mm, 16–8 mm and 8–3 mm. The effect of jigging time was also evaluated.	Concentration profile index	The coarser the material, the shorter the jigging time required to obtain an equilibrium state of stratification.
Mukherjee et al. [33]	Two-chamber Jig (plunger type)	Iron ore in the size ranges of 10–8 mm, 8–6 mm, 6–3 mm. Effect of size distribution was also evaluated.	Based on alumina content in the feed and the product	Coarser fraction separated better than the other size fractions; Reducing the size range of the feed improved separation efficiency.
Kowol and Matusiak [34]	KOMAG Jig	Gravel containing carbonate in the size ranges of 16–8 mm, 8–4 mm, 4–2 mm and 2–0.5 mm.	Concentrate and tailing content	Separation efficiency was higher in the coarser size ranges (16–8 mm and 8–4 mm) than in the finer size ranges (4–2 mm and 2–0.5 mm).
Pita and Castilho [35]	Laboratory-scale Denver Jig	Granulated plastics (1.04 to 1.37 g/cm^3) in the size ranges of 5.6–4 mm, 4–2.8 mm, 2.8–2 mm, 2–1.4 mm and 1.4–1 mm.	Products recovery	Separation efficiency increased with increasing particle size;
Crespo [18]	Laboratory-scale batch water jig	Artificial mixture of magnetite and limestone in the size ranges of 9.5–6.7 mm, 6.7–4.75 mm, 4.75–3.35 mm and 3.35–2.36 mm.	Stratification and mixing coefficients	More efficient stratification was observed when using mono-size beds composed of smaller particles;

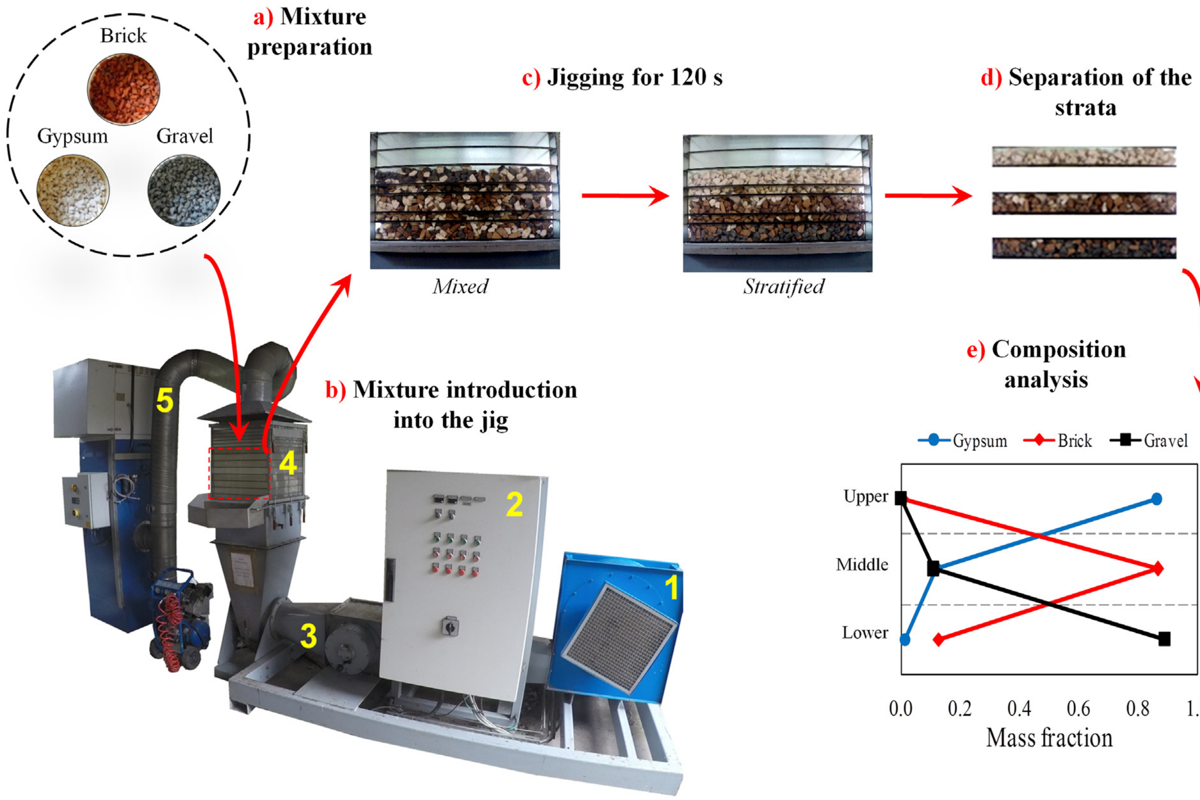

Fig. 2. Scheme of the experimental procedure for the stratification essays. The pilot-scale pneumatic jig used in the tests is composed by the following components: (1) Blower; (2) Control panel; (3) Flutter valve; (4) Jig container; (5) Dust collection hopper.

stratification efficiency. The results obtained reveals that the use of mixtures composed of particles of smaller sizes or with a wide size distribution can be surprisingly beneficial for stratification.

2. Material and methods

2.1. Equipment

The experiments were conducted in a batch pilot-scale pneumatic jig model allair® S-500 from allmineral GmbH & Co. with an operating size range of 1 to 25 mm and an average capacity of approximately 50 kg per batch (Fig. 2). During operation, the jigging bed is situated inside a chamber consisting of several overlying layers of PlexiGlass which can be removed separately, so that different vertical strata of the bed can be sampled after the sorting. This separation chamber is supported by a perforated plate (diameter of 1 mm) through which passes an upward flow of air in order to promote bed pulsation. The pulsating air flow is generated by a 15 kW blower (Combimac 49,631/B1Y1, air flow up to 73 m^3/min) and a flutter valve (rotation frequency from 0 to 300 RPM). Intake air flux and frequency can be adjusted in a control panel as a function of the percentage of the blower power (0 to 100%) and rotation of the pneumatic flutter valve, respectively. For the sake of clarity, the pulsation frequency, which is equal to the rotation frequency of the flutter valve, is denoted here in CPM units (cycles per minute).

2.2. Experimental procedure

All experiments were performed with ternary mixtures containing 37.5% of gravel, 37.5% of gypsum and 25% of brick (in bulk Vol%) (Fig. 2). The purpose to select such proportions is detailed in Section 2.3. The relative densities of each constituent, based on triplicate measures in water pycnometer, were about 2.64 g/cm^3 (± 0.00) for gravel, 2.26 g/cm^3 (± 0.02) for brick and 1.86 g/cm^3 (± 0.02) for gypsum. Intake air flux, pulsation frequency and operation time were fixed at 80% (approximately 60 m^3/min, estimated from the fan curve), 160 CPM and 120 s, respectively. These parameters were chosen based on the preliminary experimental studies of Sampaio et al. [5] and Ambrós et al. [6]. Two different experimental settings were considered in order to analyze the effect of variations in particle size on separation in pneumatic jigs. In the first set, the objective was to evaluate the effect of varying the size class of particles on bed stratification. In this case, tests were conducted with mixtures formed by near-monosized particles comprising one of the following size fractions: (i) 19.1–12.7 mm, referred as 'coarse' size; (ii) 12.7–8 mm, referred as 'intermediary' size; (iii) 8–4.75 mm, referred as 'small' size. On the other hand, the second set of tests aimed to examine the effect of expanding the particle size distribution on stratification. In this case, stratification results obtained from the coarse sized mixture (19.1–12.7 mm, or mixture M1) were compared to two other systems: one consisting of binary mixtures containing both coarse and intermediary size fractions (mixture M2) and another system composed of equal proportions of each size fraction (mixture M3) (see Table 2). Therefore, the widening of the particle size distribution was obtained by increasing the relative number of intermediary and small particles in the mixture in order to shift the particle size distribution towards the fines. In order to assure repeatability, each individual test was conducted in triplicate. Considering all test cases, the total mass of each mixture was 56 kg (± 1.5%).

2.3. Minimization of misplacement contamination

When slicing the stratified bed, each layer that compose the separation chamber is horizontally pulled away, so that particles that lie inside a given layer are discharged into a collector for subsequent composition analysis (here accomplished by hand separation of all particles contained in the layer followed by weighing individual constituents). However, given the intrusive nature of this process, misplacement of particles to the wrong layer is a common phenomenon. In order to

Table 2
Proportion (bulk Vol%) of each size fraction present in the prepared mixtures of aggregates used in the tests.

Material (density)	Size range (mm)	Proportion (bulk Vol%)				
		Coarse; M1	Intermediary	Small	M2	M3
Gypsum (1.86 g/cm^3)	19.1–12.7	33.3%	–	–	16.7%	11.1%
	12.7–8	–	33.3%	–	16.7%	11.1%
	8–4.75	–	–	33.3%	–	11.1%
Brick (2.26 g/cm^3)	19.1–12.7	33.3%	–	–	16.7%	11.1%
	12.7–8	–	33.3%	–	16.7%	11.1%
	8–4.75	–	–	33.3%	–	11.1%
Gravel (2.64 g/cm^3)	19.1–12.7	33.3%	–	–	16.7%	11.1%
	12.7–8	–	33.3%	–	16.7%	11.1%
	8–4.75	–	–	33.3%	–	11.1%
$\sum$		100.0%	100.0%	100.0%	100.0%	100.0%

quantify and so minimize the occurrence of such error, a procedure was developed to assess it. For this purpose, three large layers of the jig chamber (500 × 500 × 50) mm were each one fully filled with gravel (lower layer), brick (middle layer) and gypsum (upper layer) as shown in Fig. 3(a), all components in the size range of 19.1–12.7 mm. Then, each layer was carefully removed and the composition of each stratum was determined. Two types of interlayer contamination were identified, referred as contamination by *percolation* and contamination by *pulling*. The first one involves the percolation of particles from upper to lower layers due to the disruption caused by the slicing of the bed. The second one is related to the pulling of particles situated at the interface with an upper layer that is being removed off the system. It was possible to verify that these contaminations generated variations (in content and recovery in each layer) as high as 8.4% among tests under identical conditions. In such circumstances, examining discrete variations in composition caused by variations in particle size only could be impracticable.

Aiming to enhance repeatability over the tests, the layers distribution was modified as shown in Fig. 3(b). In this case, a smaller layer (500 × 500 × 25) mm of Plexiglass was positioned between the original layers in order to absorb the fluctuations caused by the slicing action. The product contained within these layers were not considered in the subsequent composition analysis. By means of this procedure, the average variation in composition among tests under identical conditions was reduced from 6.3% to 0.8% in the lower layer and from 8.4% to 0.0% in the upper layer. Therefore, the layers configuration showed in Fig. 3(b) was the one adopted for the experiments. It should be noted, however, that such configuration was not able to reduce the contamination by slicing in the middle layer. Thus, in the following discussion, the middle layer and its main constituent, brick, were not considered as products (concentrated) but only as intermediary or middling elements within the system. Nevertheless, for the sake of segregation analysis, the content of the middle layer as well as brick distribution over the bed was also examined.

2.4. Analysis

Further to the evaluation of the stratification in terms of grade and recovery of products, a stratification index was introduced in order to quantify the separation efficiency, given by:

$$I_S = \frac{\sigma_{experimental}}{\sigma_{stratified}} \tag{2}$$

where:

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{k} F_i \left[\sum_{j=1}^{n} \left(X_{ij} - \overline{X}\right)^2\right] \tag{3}$$

$$\overline{X_i} = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \tag{4}$$

where I_S is the stratification index, $X_{ij}$is the concentration of constituent i ($k = 1,2,3$) contained within layer j, $\overline{X_i}$ is the average concentration of the constituent i in the n stratum ($n = 1,2,3$), F_i is the overall fraction of the component i in the sampled strata and σ^2 is the weighted mean of the variances of components concentration in the n sampled strata. The proposed index is based on the indexes used by Fan et al. [36] for calculation of the degree of mixing in multi-component systems, where $\sigma = 0$ for the case of a perfectly homogeneous system, while its value for the case of ideal stratification depends on the number of components within the system as well as their mass or volumetric proportions. Thus, the stratification index consists in the ratio between the experimental variance and the variance for the case of ideal stratification. Consequently, $I_S = 0$ corresponds to the perfectly mixed state while $I_S = 1$ corresponds to the perfectly stratified state of the system.

In the present case, the index was determined considering only the concentration of gypsum and gravel in their predominant strata versus

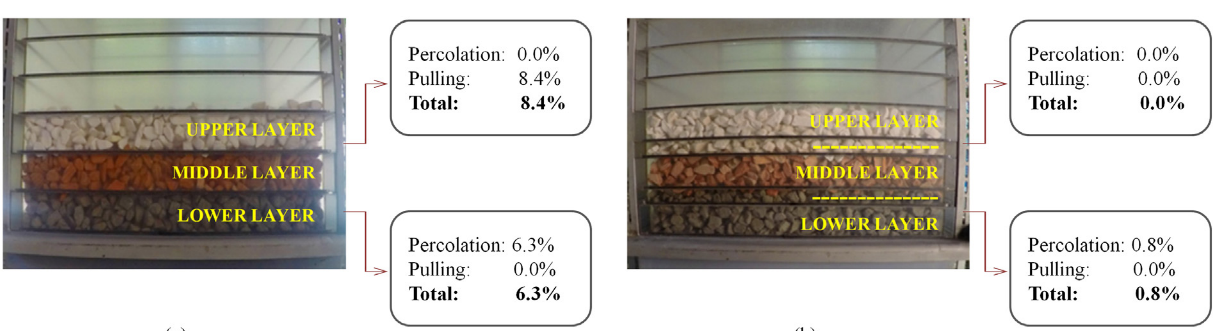

Fig. 3. Analysis of interlayer contamination related to misplacement errors during the extraction of products. (a) Typical configuration of overlapped layers; (b) Modified configuration by using intermediary layers that absorb misplacement errors. The values refers to the essay performed with the mixture M1 (19.1–12.7 mm).

their concentration in the rest of the system ($n = 2$ and $k = 2$). This choice partially reduces possible errors on the index that may be caused by misplacement contaminations in the middle layer (see Section 2.3). The value of $\sigma_{stratified}$, obtained by considering perfect stratification in Eq. (3), varied slightly in each case due to differences in overall proportion in the sampled zones, corresponding to an average value of 0.60 ($\pm$ 0.01) for all tested cases.

3. Results

In all cases, the overall stratification pattern consisted in the division of the bed in three distinct strata: a lower stratum filled with large quantities of gravel; an intermediary layer preferably containing brick particles; and an upper layer almost entirely composed of gypsum particles. This pattern was already expected beforehand on the basis of prior work developed by our group [4,5,21]. The following sections present in detail the influence that variations in particle size and size distribution of the bed have had on this general stratification profile. However, since the occurrence of experimental artifacts was a recurrent concern during tests (see Section 2.3), it is suitable at first to examine the influence of the sampling configuration adopted (Fig. 3.b) on the repeatability of separation results.

3.1. Assessment of the misplacement contamination

The influence of misplacement contamination was assessed by analyzing the standard deviation (SD) of the content and recovery measurements of the main constituent in each sampled stratum for each test in triplicate (Table 3), given by:

$$SD = \sqrt{\frac{\sum \left(X_{i,j} - \overline{X}_{i,j}\right)^2}{3}} \tag{5}$$

where $\overline{X}_{i,j}$ is the average concentration of constituent i contained within layer j measured for the three tests. It can be observed that for upper and lower strata, SD levels were lower than ±3% in almost all cases, with notable exception for a relative high value (± 4.09%) for recovery in the lower layer in the mixture of size range 12.7–8 mm. Also, as a rule, SD values were higher in the lower stratum than in the upper one, which can be related with the occurrence of contamination by percolation. Similarly, the highest SD values observed in the middle layer may be derived from the existence of both types of contamination, i.e. by percolation and by pulling, giving rise to greater discrepancy among composition measurements than those found in top and bottom layers.

3.2. Effect of varying particle size class

The effect of varying the size class of the bed particles is schematically shown in Fig. 4. As a general rule, separation efficiency increased as the particle size decreased in such way that the stratification index for the system containing small particles (8–4.75 mm) was 28% greater than for that containing coarse particles (19.1–12.7 mm). More detailed information about changes generated by varying the particle size is illustrated by the stratification profiles (mass content of each material in each stratum) shown in Fig. 4(a). In the lower layer, the mass content of gravel increased significantly (7%) when the coarse sized bed was changed by a bed containing particles of intermediary size and increased slightly (3%) for the system containing small particles. On the other hand, decreasing the particle size from intermediary to the small size class resulted in lower gypsum content in the upper stratum. Also, content variation in the outer strata (lower and upper layers) was no greater than 10%, whereas content variation of brick in the middle layer was about of 40% in mass. Considering the three mixtures, the maximum deviation of the measured contents were of ±1.24% in mass in upper and lower strata and ± 6.73% in mass in the middle stratum, confirming the predicted greater extent of contamination in this layer.

The distribution profiles (mass recovery of each material in each stratum) exhibited in Fig. 4(b) provide a more detailed insight into differences in local concentration. On the whole, the results seem to indicate that the use of beds of smaller sizes can have resulted in a more compact rearrangement of particles in the stratified state. Particularly, there was a remarkable tendency to increase the concentration of both the lighter and the denser particles (gypsum and gravel, respectively) in deeper layers of the bed when decreasing the size class of the system. This can be noted by the increasing clustering of gravel in the lower stratum together with the increasing concentration of gypsum in the middle stratum. On the other hand, a partially opposite tendency can be observed for brick, which decreased its concentration in the lower stratum while increased it in the middle stratum as bed composition vary from coarse size to small size particles. Nevertheless, it is reasonable to suppose that such behavior can represent a larger upward displacement of brick granules by the heavier gravel particles in beds composing by particles of smaller sizes.

3.3. Effect of varying particle size distribution

The overall response to the widening of the size distribution was similar to that observed when decreasing the absolute particle size of the bed (Section 3.1). As can be seen from Fig. 5, mixtures composed by larger size distributions showed higher segregation levels, with an overall increase in the stratification index of 26%. Once again, an increase in gravel content in the lower stratum was accompanied by a little decrease in gypsum content in the upper stratum, while the content of brick in the middle layer increased substantially. Also, the effect of increasing the fraction of smaller particles within the system was very similar to that of decreasing the size class, as can be seen in the distribution profiles showed in Fig. 5(b). In the main, a greater recovery of gravel and brick were obtained when increasing size distribution while gypsum recovery decreased slightly, suggesting that the use of multi-sized mixtures resulted in an increase of bed compaction after jigging. This trend was expected beforehand since wide size distributions allow the particles to rearrange into denser packing structures [23,27].

The individual recovery patterns of each size range are displayed in Fig. 6. On the whole, the increase of dispersion in particle size enhanced the concentration of large particles in deeper layers of the bed (namely, middle and lower layers) and, on the other hand, increased the concentration of small particles in the top layer. In other words, it contributed to a better concentration of heavy large particles and light small particles, having the opposite effect on heavy small and light large particles. As detailed in Fig. 6(a), the amount of coarse gypsum recovered in the upper layer gradually decreased as the size distribution increased, while the opposite occurred for coarse particles of brick and gravel in their respective layers. On the other hand, contrary behavior was observed for small particles (Fig. 6.c), in which the increase in size distribution improved gypsum recovery in the upper layer, but impaired

Table 3
Standard deviation values of the content and recovery measurement of the main component contained in each sampled stratum.

Mixture size range	Measures	Strata		
		Upper	Lower	Middle
19.1–12.7 mm (coarse fraction)	Content	± 0.36%	± 0.58%	± 4.37%
	Recovery	± 0.48%	± 2.93%	± 1.71%
12.7–8 mm (Inter. fraction)	Content	± 0.61%	± 2.19%	± 6.73%
	Recovery	± 0.81%	± 4.09%	± 4.77%
8–4.75 mm (Small fraction)	Content	± 1.00%	± 1.24%	± 2.15%
	Recovery	± 1.33%	± 1.10%	± 2.50%
19.1–8 mm (mixture M2)	Content	± 0.03%	± 2.00%	± 1.26%
	Recovery	± 0.82%	± 1.58%	± 2.93%
19.1–4.75 mm (mixture M3)	Content	± 0.49%	± 1.01%	± 2.58%
	Recovery	± 2.15%	± 2.05%	± 1.73%

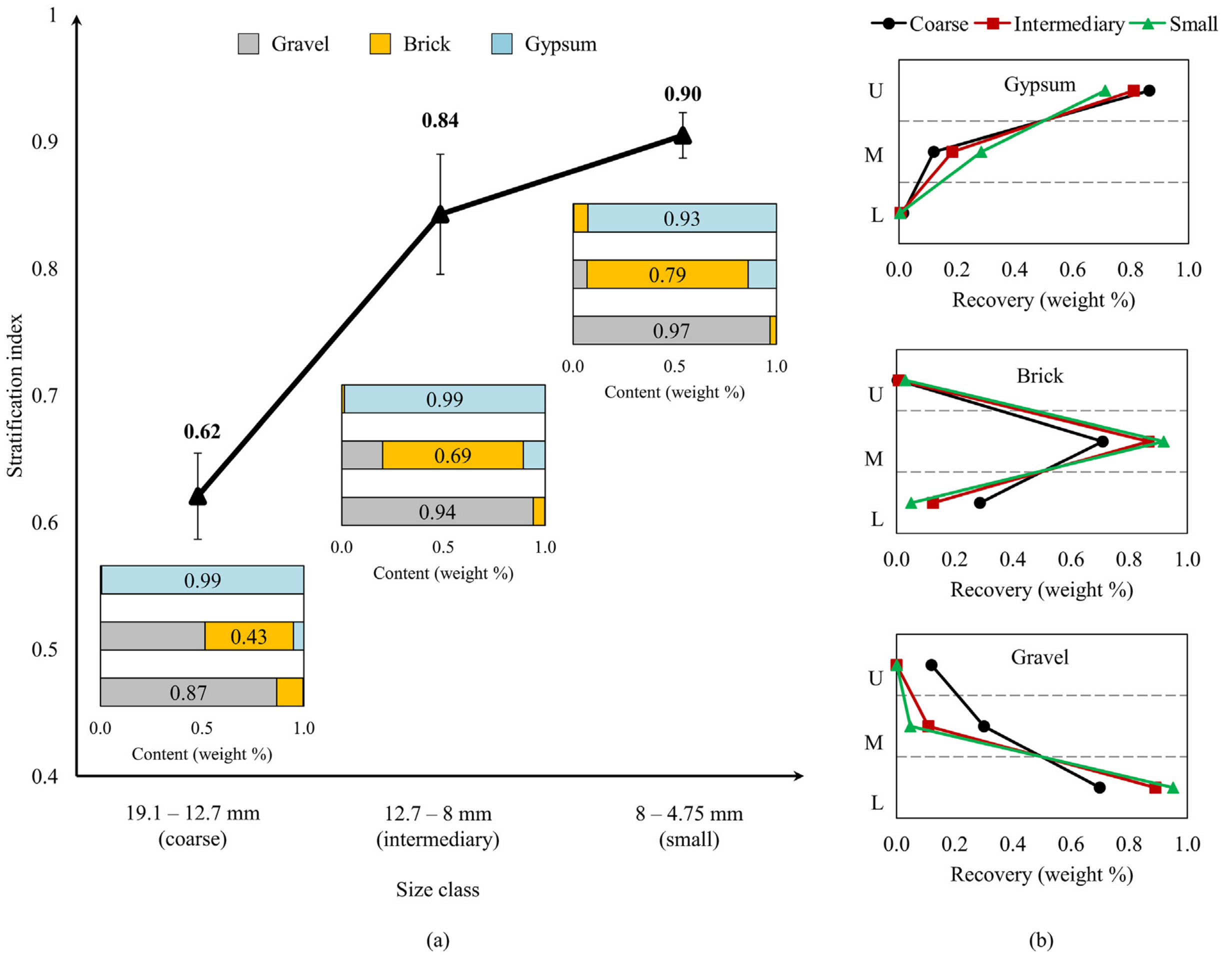

Fig. 4. (a) stratification indexes, stratification profiles (mass content in each stratum) and (b) distribution profiles (mass recovery in each stratum) resulting from tests with mixtures containing different size classes. U – upper stratum; M – middle stratum; L – lower stratum.

concentration of brick and gravel in the other strata. Finally, the increase of size distribution had only a marginal effect on segregation of intermediary size particles. As exhibited in Fig. 6(b), the segregation pattern of gypsum was almost the same in all cases, while a sensible improvement of gravel concentration in the lower stratum could be observed. Also, the segregation pattern of brick suggests that larger size distributions played a role in increasing brick concentration in the top layer.

From a practical standpoint, the results points to the possibility that a greater recovery could be obtained in the dense product as fractions of smaller particles are included in the feed, which could be removed from the concentrate by screening just after the jigging stage, if necessary. On the other hand, adding a fraction of larger particles in the feed could have a similar effect on the recovery of the light product. However, experimental evidences underpinning this assumption only for systems in which different particle size classes have similar volumetric fractions. Also, the effect of post-classification of the concentrates on their mass content should be assessed. Figs. 7 and 8 illustrate it for the cases of mixtures M2 and M3, respectively. In both cases, for the hypothetical case in which the dense product is split into different size classes, the variations in product content among these fractions and between them and the non-classified product, where they exist, are negligible. Thus, in operating with a larger size range (19.1–8 mm in mixture M2; 19.1–4.75 mm in mixture M3) it would be possible to achieve a much higher overall mass recovery rate of denser product (+20% for M2 and + 25% for M3, see distribution profiles in Fig. 5.b) than in operating with the coarse fraction only, with the additional benefit of a reasonably higher mass content in the final product. On the other hand, no significant gain in the recovery or content of dense product would be obtained in comparison to processing the intermediary or the smaller size fractions only.

The same trend can be also observed when splitting the light product of mixture M2 in their constituent size classes, with exception by a slight decrease in gypsum recovery (≈ 5% in mass, see Fig. 5.b) in comparison to that of mixture M1. However, a decrease in gypsum content of about 10% in mass can be observed when comparing the small size range (8–4.75 mm) in mixture M3 with the other size fractions, whereas mass recovery in this fraction was relatively higher (Fig. 6). Thus, contrary to observed in the dense product, the inclusion of intermediary size particles had no significant effect on mass recovery and content of the light concentrate, but the inclusion of particles of smaller sizes negatively affected the overall content of the light product. Exceptionally, the enlargement in size distribution would be somewhat advantageous for the light concentrate in mixture M3 when compared to the mixture containing small particles only, which showed a relative overall recovery level of gypsum about 9% lower (but an overall content 3% higher) than mixture M3.

4. Discussion

The obtained results corroborate and extend to pneumatic jigs an already accepted trend about the effect of varying particle size in jigging beds: that larger particles tend to concentrate in the lower zones of the bed, whereas smaller particles tend to concentrate in upper zones when jigging mixtures containing particles of different size classes [25,38,39]. On the other hand, regarding the effect of particle size and size distribution on density stratification, the obtained findings contrast

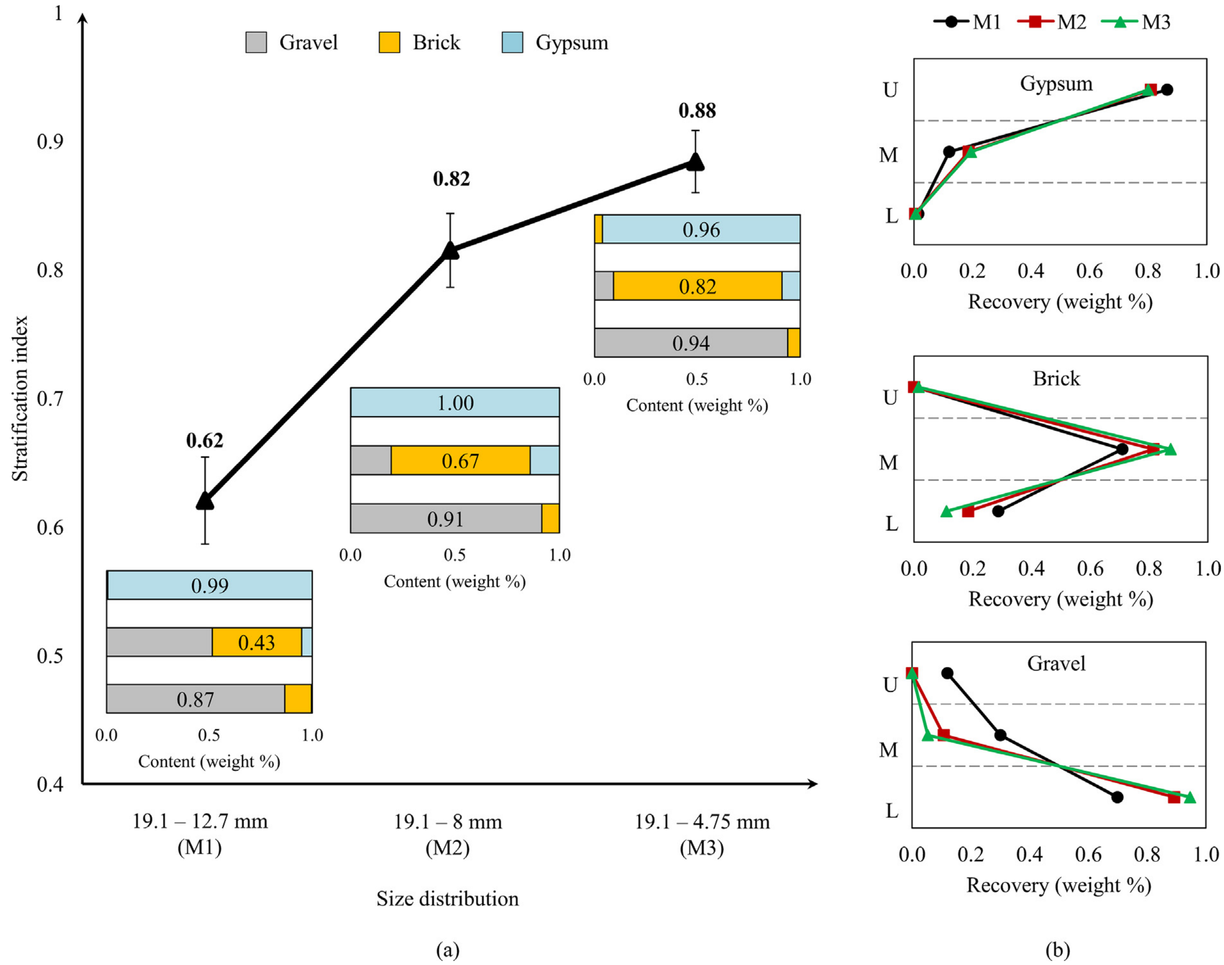

Fig. 5. (a) stratification indexes, stratification profiles (mass content in each stratum) and (b) distribution profiles (mass recovery in each stratum) resulting from tests with mixtures containing different size distribution. U – upper stratum; M – middle stratum; L – lower stratum.

with the usual trend reported for hydraulic jigs, which indicates that stratification efficiency increases with increasing particle size of the system (see Table 1). As previously reported, experimental results revealed that the smaller the size of particles within the bed, the better was the stratification efficiency. Also, the stratification level increased when jigging mixtures contained larger size ranges. Despite a deep analysis of the stratification mechanisms behind such phenomena being beyond the scope this study, some experimental evidences can be discussed in the light of the particularities involved in pneumatic jigging together with literature data.

Despite being based on the same basic mechanisms, for being an air-based separator the pulsating conditions in pneumatic jigs are somewhat different from that of hydraulic jigs. The most obvious difference is that, in contrast to water, the density of air is insignificant in relation to the solid particles of the bed. Thus, in order to compensate this discrepancy of densities, much higher fluid velocities must be employed in order to reach the required drag force to lift the bed. Since the required fluid velocity increases with particle size and also must be enough to lift all the bed, the air velocities used to elevate coarse particles can be much higher than those necessary to elevate existing small particles, giving rise to segregation by size due to the displacement of the smallest particles towards the top of the bed [23,37].

This phenomenon is probably intensified in pneumatic jigs due to two inherent operational features of the equipment. Firstly, pneumatic jigs usually employ a constant upward air flow together with the pulsating flow in order to keep the bed open and thus facilitate stratification [40], which can restrain the downward motion of particles, especially the lighter smaller ones. Secondly, the suction stroke is virtually absent, since the air passing through the bed does not return to the system. Consequently, it is reasonable to suppose that percolation trickling has a minor role in comparison to conventional hydraulic jigs. All these factors lead to a higher tendency to displace to and even eject small particles from the surface of the bed during jigging operation.

An illustration of such a trend is displayed in Fig. 9, where the motion of the partially stratified bed of mixtures M1 (coarse particles only) and M3 (particles of mixed sizes) are compared at different times of the separation. As can be noted, the bed motion during approximately the same duration of a jigging cycle (bed expansion and contraction) showed significantly different patterns in each mixture. Whereas the bed in mixture M1 lifted in a approximately uniform manner, the bed elevation in mixture M3 was usually turbulent, being noticeable the larger amplitude of motion of the smallest particles of gypsum located in the top in such a way that some of them were even ejected out the bed surface. During the contraction phase, the bed of mixture M1 settled as a whole on the jig screen, whereas in mixture M3 it remained partially disperse so that many small particles remained fluidized above the bed. As a result, during jigging of mixture M3 the upper part of the bed moved in a seemingly transient manner, being sometimes difficult even identify visually the phase of the jigging cycle.

Since the upward air flow was kept constant in all tests, it may be assumed that the observed unstable behavior of bed motion can be

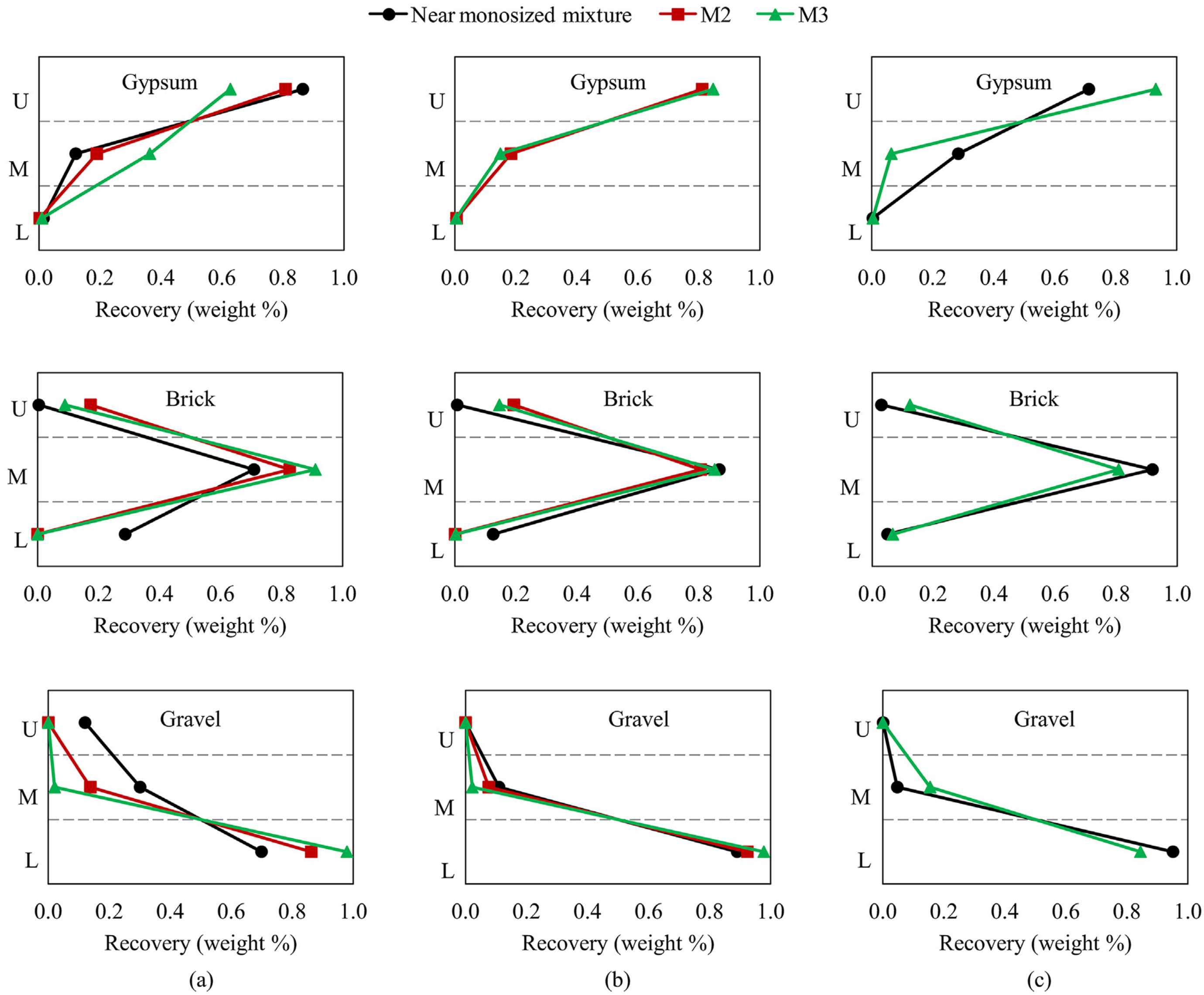

Fig. 6. Recovery profiles of each size class along bed strata. (a) Coarse particles. (b) Intermediary particles. (c) Small particles.

attributed to the comparatively higher drag force acting on existing small particles within the system. This is consistent with results obtained by Formisani [41], which observed a lowering of the minimum fluidization velocity when increasing the volumetric fraction of fine particles in a binary mixture. Mukherjee and Mishra [42] also observed that the finer size class controlled the separation efficiency when the jig feed (Denver type) had a higher proportion of fines (about 50% in mass), since the particles experienced a higher drag due to the lesser voidage of the bed during pulsation. In the current case, the differential fluid drag on particles of varied sizes may have increased the void fraction of the dispersed bed, thus allowing coarse particles to sink more easily in comparison to the system containing coarse particles only, resulting in a greater concentration of heavy, coarse particles in the dense product and light, coarse particles in the deeper strata. Also, it may have increased the removal of undesired light small particles from the dense product, whereas, on the other hand, it increased the contamination of the light product by fine dense particles. These trends are in line with the previously described results (see Fig. 6).

Similarly, the core concept of the potential energy theory of jigging [19] is useful in describing some experimental evidences. From its thermodynamic viewpoint, the decreasing of potential energy caused by the bed rearrangement is the driving force behind the stratification. This reduction of energy manifests in the form of a lowering of the center of gravity of the granular bed in such a way that the final segregation pattern will be the one that promote a higher compaction of the jig bed. Results in line with this mechanism were also reported in most recent studies involving particle segregation in vibrated systems [43] and fluidized beds [44]. In this context, the observed variation of stratification performance may have been influenced by modifications of the bed compaction due to changes in particle size and size distribution.

In order to examine the existing connection among particle size, bed packing and stratification efficiency, the packing density (solid volume in a unit total volume) of the lower layer was compared to the stratification indexes obtained in all tested conditions, as shown in Fig. 10. The x-axis represents the ratio between the average particle size of the system and the container size (d_p/L, where L is the length of the jig container, equal to 500 mm). The reason to adopt such notation will be subsequently made clear. It is possible to observe that stratification index and packing density increased in unison when using beds composed of smaller particles or by wider size distributions. On the basis of the potential energy theory, it can be inferred that beds consisting of smaller particles or by wide distributions in the particle size ought to be subjected to higher decrease in the absolute potential energy of

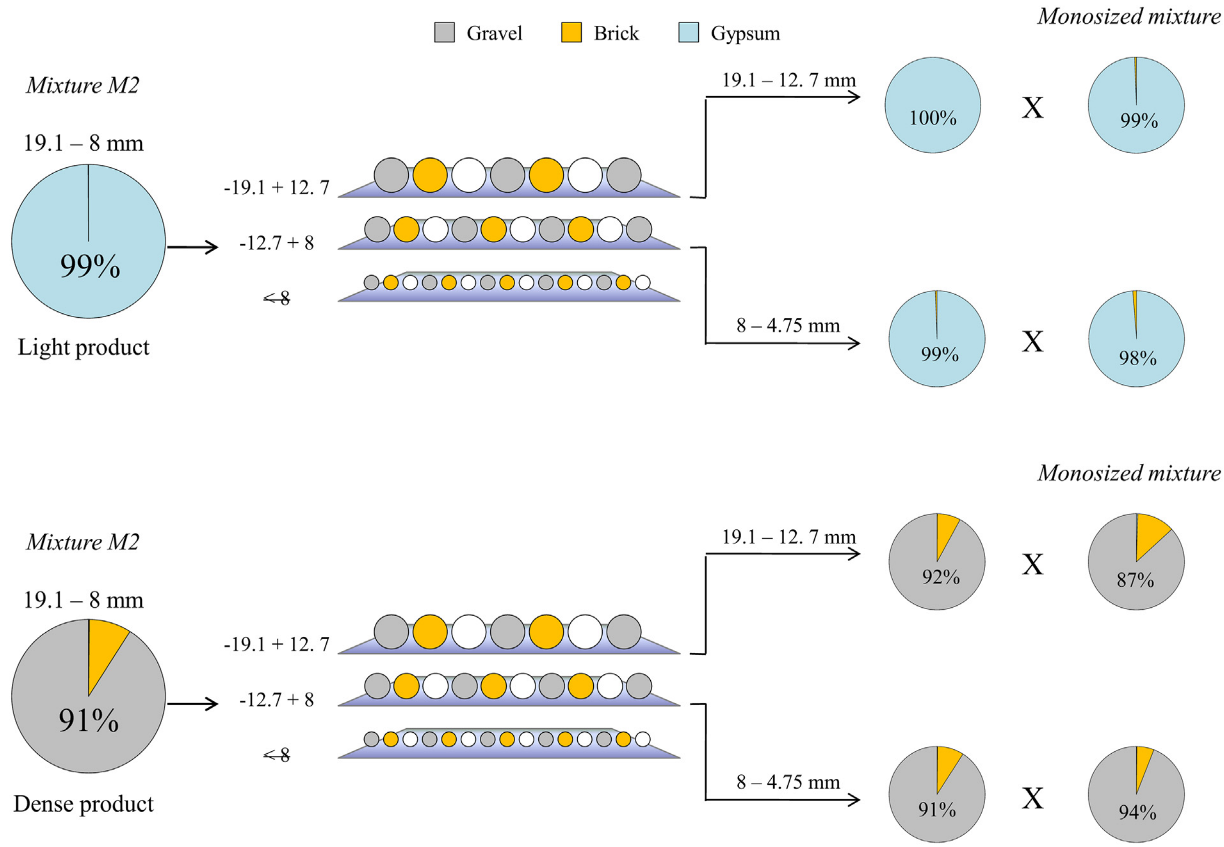

Fig. 7. Comparison between the overall content (weight%) of mixture M2, the content (weight%) of the classified jigging products and the content (weight%) obtained when jigging monosized mixtures of the same size class.

the system (i.e. become more compact), which in the cases studied has resulted in a better particle stratification.

However, although a greater packing should be expected to systems containing larger distribution in particle size [24,28], the same is not necessarily expected to occur a priori in function of the mere decrease in the absolute size of the particles in a given system. In the classic study on the link between particle size and packing density, Sohn and Moreland [45] reported that the latter is independent of the nominal size of particles, but only of the particle size distribution within the system. Nevertheless, some light is shed on this behavior in the recent study of Zhao et al. [46], which demonstrated that the packing density of a granular system subjected to vertical vibration is affected by the ratio between the particle size and the container size, such as represented in Fig. 10. Particularly, the larger the container size in relation to the particles of the system, the greater tends to be the final packing density of the system. Packing disturbances caused by wall effects have been pointed out as the main reason for such phenomenon, which is in very good agreement with previous studies developed by our group in the case of pneumatic jigs [21]. Therefore, it may be reasonable to suppose that the use of smaller bed particles increases the relative size of the container, resulting in a higher compaction of the stratified system.

It is important to emphasize that other phenomena can influence the observed results. For instance, results obtained by Metzger et al. [29] and Jain et al. [30] suggest that introducing intermediate particle sizes in between the largest and the smallest in the original mixture can reduce the tendency of size segregation in granular systems. In the present case, the presence of the intermediary size fraction (12.7–8 mm) in mixture M3 may have helped to inhibit size segregation in the system, while it has had little or no impact on segregation by density. However, examining such phenomenon is beyond the scope of this study.

5. Conclusions

Particle size variation is a key contributor to particle separation in jigs and despite much research in the area, very little attention has been given to the case of pneumatic jigging. In the current work, the influence of varying the particle size and the size distribution on particle stratification in a pilot-scale pneumatic jig was evaluated. The bed composition after several tests with ternary mixtures of different sizes has been examined in detail and an analytic expression for the estimation of the stratification efficiency has been proposed. Under the tested conditions, our results provide compelling evidence that stratification by density can be enhanced when using particles of smaller sizes or when widening the particle size range of the system by means of shifting the size distribution towards the small particles.

Similarly, it has been found that the use of mixtures containing particles of different sizes contributed to increase the recovery of heavy large particles in the dense product and light small particles in the light product. On the other hand, the decrease of the bed particle size and the expansion of the particle size distribution skewed towards the small particles increase contamination of heavy small particles in the light product. From the operational standpoint, the results consider the possibility that a greater recovery and content of the coarse, dense product could be obtained through the inclusion of smaller particles in the feed, which can be easily removed by screening after jigging. On the other hand, no significant gain in recovery or content would be obtained when compared to the jigging of mixtures containing smaller size particles only.

The obtained results were discussed in the light of the particularities involved in pneumatic jigging operation together with the existing differences in bed packing derived from the variation of particle size within the system. The combination of differential drag on the particles of varied sizes, the absence of the suction stroke and the existence of a

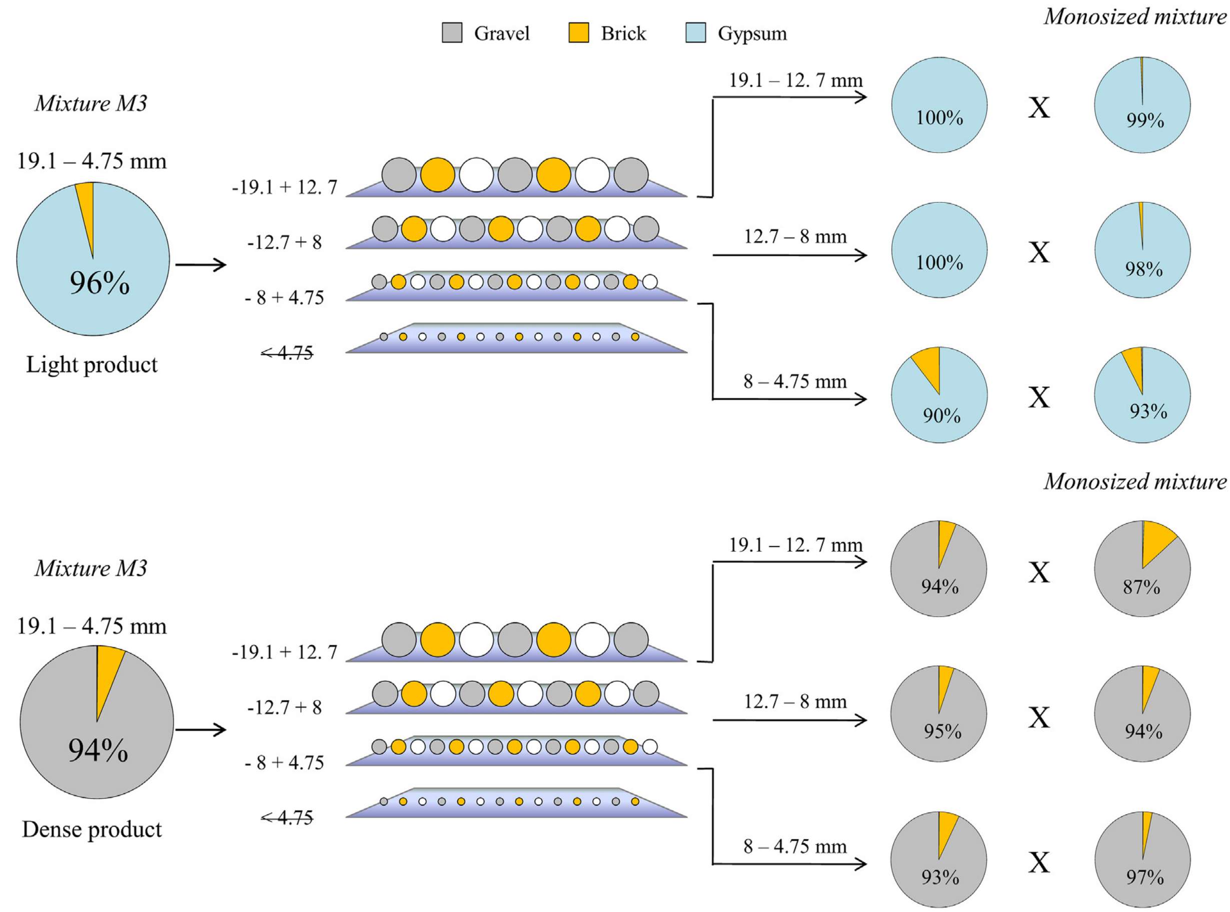

Fig. 8. Comparison between the overall content (weight%) of mixture M3, the content (weight%) of the classified jigging products and the content (weight%) obtained when jigging monosized mixtures of the same size class.

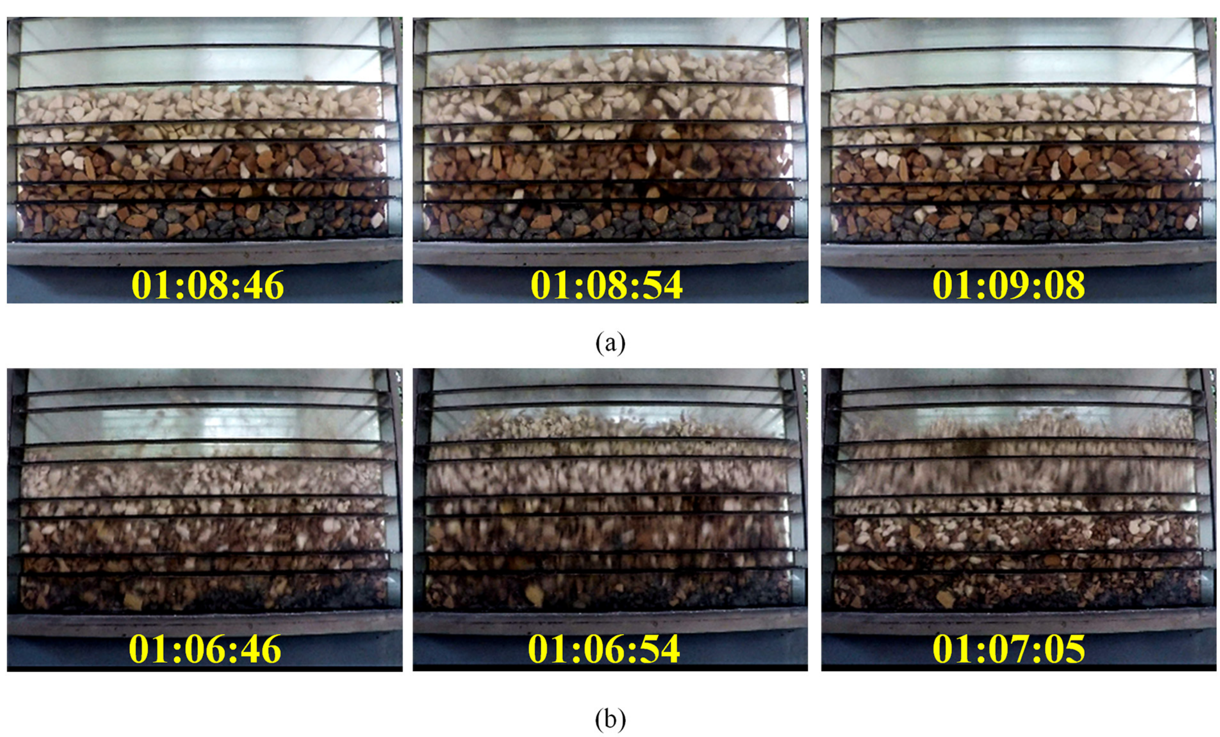

Fig. 9. Snapshots of bed motion when jigging mixtures M1 (a) and M3 (b).

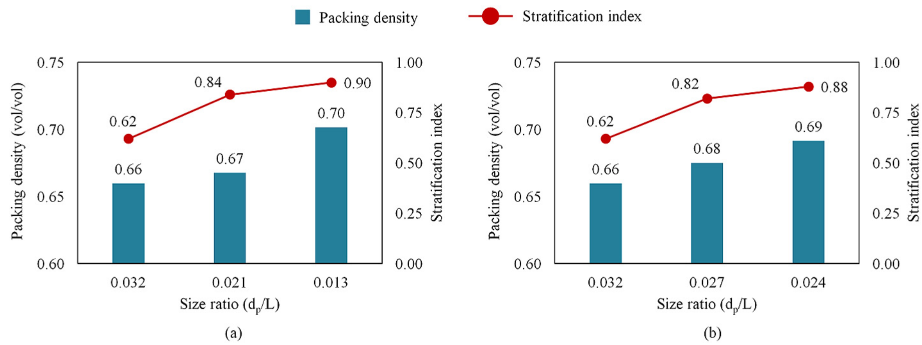

Fig. 10. Packing density of the lower layer and stratification index variation in function of the size ratio of the system, dp/L (where L is the length of the jig container, equal to 500 mm). (a) Variation of particle size. (b) Variation of size distribution.

constant upward air flow seem to benefit all together the concentration of smaller particles in the top of the bed and coarse particles in the bottom. In parallel, mixtures formed by particles of smaller sizes or by wider size distributions tend to produce more compact beds, thus leading to greater recovery of the dense product.

It is important to emphasize that the obtained results correspond to a limited range of conditions. Full experimental data was only obtained from mixtures containing similar volumetric fractions of each constituent, and although the effect of particle shape has not been addressed experimentally, it can be assumed to exist. Also, pulsating parameters were kept constant in all cases. Future studies should be undertaken to evaluate the effect of expanding differences in particle size within the system and also working with different operating conditions, number of constituents and different particle densities and shapes. A deeper understanding of particle stratification in pneumatic jigs and developing ways to optimize it can be of great benefit for future applications of the technique.

Acknowledgements

Dr. Simões dos Reis is grateful to the Council for the Development of Higher Education at Graduate Level, Brazil (CAPES) for the postdoctoral scholarship granted through National Postdoctoral Program (PNPD).

References

[1] R. Weinstein, R. Snoby, Advances in dry jigging improves coal quality, Min. Eng. 1 (2007) 29–34.
[2] C.H. Sampaio, W. Aliaga, E.T. Pacheco, E. Petter, H. Wotruba, Coal beneficiation of Candiota mine by dry jigging, Fuel Process. Technol. 89 (2008) 198–202, https://doi.org/10.1016/j.fuproc.2007.09.004.
[3] F. Boylu, E. Talı, T. Çetinel, M.S. Çelik, Effect of fluidizing characteristics on upgrading of lignitic coals in gravity based air jig, Int. J. Miner. Process. 129 (2014) 27–35, https://doi.org/10.1016/j.minpro.2014.04.001.
[4] B. Cazacliu, C.H. Sampaio, G. Miltzarek, C. Petter, L. Le Guen, R. Paranhos, F. Huchet, A.P. Kirchheim, The potential of using air jigging to sort recycled aggregates, J. Clean. Prod. 66 (2014) 46–53, https://doi.org/10.1016/j.jclepro.2013.11.057.
[5] C.H. Sampaio, B.G. Cazacliu, G.L. Miltzarek, F. Huchet, L. Le Guen, C.O. Petter, R. Paranhos, W.M. Ambrós, M.L. Silva Oliveira, Stratification in air jigs of concrete/brick/gypsum particles, Constr. Build. Mater. 109 (2016) 63–72, https://doi.org/10.1016/j.conbuildmat.2016.01.058.
[6] W.M. Ambrós, C.H. Sampaio, B.G. Cazacliu, G.L. Miltzarek, L.R. Miranda, Usage of air jigging for multi-component separation of construction and demolition waste, Waste Manag. 60 (2017) 75–83, https://doi.org/10.1016/j.wasman.2016.11.029.
[7] Z. Wang, P. Hall, N.J. Miles, T. Wu, P. Lambert, F. Gu, The application of pneumatic jigging in the recovery of metallic fraction from shredded printed wiring boards, Waste Manag. Res. 33 (2015) 785–793, https://doi.org/10.1177/0734242X15589782.
[8] M.A. Abd Aziz, K. Md Isa, R. Ab Rashid, Pneumatic jigging: Influence of operating parameters on separation efficiency of solid waste materials, Waste Manag. Res. 35 (2017) 647–655, https://doi.org/10.1177/0734242X17697815.
[9] C.H. Sampaio, L.M.M. Tavares, Beneficiamento gravimétrico: Uma introdução aos processos de concentração mineral e reciclagem de materiais por densidade, Editora UFRGS, Porto Alegre, 2005.
[10] G.J. Lyman, Review of jigging principles and control, Coal Prep. 11 (1992) (1992) 145–165.
[11] S. Cierpisz, M. Kryca, W. Sobierajski, Control of coal separation in a jig using a radiometric meter, Miner. Eng. 95 (2016) 59–65, https://doi.org/10.1016/j.ifacol.2015.10.081.
[12] A. Laplante, S. Gray, Advances in Gravity Gold Technology, Developments in Mineral Processing, vol. 15, 2005 280–307.
[13] B.A. Wills, J.A. Finch, Wills' Mineral Processing Technology: An Introduction to the Practical Aspects of Ore Treatment and Mineral Recovery, 8th ed. Butterworth-Heinemann, Oxford, 2016.
[14] H. Kirchberg, W. Hentzschel, A study of the behavior of particles in jigging, International Mineral Dressing Congress, Stockholm, Almqvist and Wiksell, 1958 193.
[15] G.D. Lill, H.G. Smith, A Study of the Motion of Particles in a jig Bed. International Mineral Processing Congress, Institution Of Mining And Metallurgy, London, 1960 515–535.
[16] A.M. Gaudin, Principles of Mineral Dressing, Mcgraw-hill, Nova York, 1939.
[17] S. Viduka, Y. Feng, K. Hapgood, P. Schwarz, CFD–DEM investigation of particle separations using a sinusoidal jigging profile, Adv. Powder Technol. 24 (2013) 473–481.
[18] E.F. Crespo, Modeling segregation and dispersion in jigging beds in terms of the bed porosity distribution, Miner. Eng. 85 (2016) 38–48, https://doi.org/10.1016/j.mineng.2015.10.012.
[19] Mayer, F.W., 1964. Fundamentals of Potential Theory of the Jigging Process, Proceedings of the 7th International Mineral Processing Congress, New York, Part 2, pp. 75–86.
[20] L.M. Tavares, R.P. King, A useful model for the calculation of the performance of batch and continuous jigs, Coal Prep. 15 (1995) 99–128, https://doi.org/10.1080/07349349508905291.
[21] W.M. Ambrós, B.G. Cazacliu, C.H. Sampaio, Wall effects on particle separation in air jigs, Powder Technol. 301 (2016) 369–378, https://doi.org/10.1016/j.powtec.2016.06.034.
[22] A.F. Taggart, Handbook of Mineral Dressing: Ores and Industrial Minerals, John Wiley & Sons, Austin, 1945.
[23] J.F. Richardson, J.H. Harker, J.R. Backhurst, Chemical Engineering: Particle Technology and Separation Processes, 5th ed. Butterworth-heinemann, Oxford, 2002.
[24] A.D. Rosato, Y. Lan, D.T. Wang, Vibratory particle size sorting in multi-component systems, Powder Technol. 66 (1991) 149–160, https://doi.org/10.1016/0032-5910(91)80096-2.
[25] L.C. Woollacott, M. Silwamba, An experimental study of size segregation in a batch jig, Miner. Eng. 94 (2016) 41–50, https://doi.org/10.1016/j.mineng.2016.04.003.
[26] M. Shapiro, V. Galperin, Air classification of solid particles: a review, Chem. Eng. Process. 44 (2005) 279–285.
[27] D. Chen-Long, H. Ya-Qun, Z. Yue-Min, H. Jing-Feng, W. Bao-Feng, Development and application of the active pulsing air classification, Procedia Earth Planet. Sci. 1 (2009) 667–672.
[28] M. Schröter, S. Ulrich, J. Kreft, J.B. Swift, H.L. Swinney, Mechanisms in the size segregation of a binary granular mixture, Phys. Rev. E - Stat. Nonlinear, Soft Matter Phys. 74 (2006)https://doi.org/10.1103/PhysRevE.74.011307.
[29] M.J. Metzger, B. Remy, B.J. Glasser, All the Brazil nuts are not on top: Vibration induced granular size segregation of binary, ternary and multi-sized mixtures, Powder Technol. 205 (2011) 42–51, https://doi.org/10.1016/j.powtec.2010.08.062.
[30] A. Jain, M.J. Metzger, B.J. Glasser, Effect of particle size distribution on segregation in vibrated systems, Powder Technol. 237 (2013) 543–553, https://doi.org/10.1016/j.powtec.2012.12.044.
[31] O. Olajide, E.H. Cho, Study of the jigging process using a laboratory-scale Baum jig, Miner. Metall. Process. 4 (1987) 11–14.
[32] R.X. Rong, G.J. Lyman, The effect of jigging time and air cycle on bed stratification in a pilot scale Baum jig, Fuel 71 (1992) 115–123.
[33] A.K. Mukherjee, D. Bhattacharjee, B.K. Mishra, Role of water velocity for efficient jigging of iron ore, Miner. Eng. 19 (2006) 952–959, https://doi.org/10.1016/j.mineng.2005.10.023.
[34] D. Kowol, P. Matusiak, Badania Skutecznosci Osadzarkowego Oczyszczania Kruszywa z Ziaren Weglanowych, Min. Sci.: Miner. Aggreg. 22 (2015) 83–92.
[35] F. Pita, A. Castilho, Influence of shape and size of the particles on jigging separation of plastics mixture, Waste Manag. 48 (2016) 89–94, https://doi.org/10.1016/j.wasman.2015.10.034.
[36] J.R. Too, R.M. Rubison, L.T. Fan, F.S. Lai, Studies on multicomponent solids mixing and mixtures: part III mixing indices, Powder Technol. 24 (1979) 73–89.

[37] M. Rhodes, Introduction to Particle Technology, John Wiley & Sons, Chichester, 2008.
[38] L.M. Tavares, Monte carlo simulations on the potential energy theory of jigging, Coal Prep. 20 (1999) 71–83, https://doi.org/10.1080/07349349908945593.
[39] Y. Xia, F.F. Peng, E. Wolfe, CFD simulation of fine coal segregation and stratification in jigs, Int. J. Miner. Process. 82 (2007) 164–176, https://doi.org/10.1016/j.minpro.2006.10.004.
[40] Allmineral Aufbereitungstechnik GmbH & Co, , allair® http://www.allmineral.com/fileadmin/user_upload/allmineral/pdf/allair.pdf 2018, Accessed date: 21 March 2018.
[41] B. Formisani, Packing and fluidization properties of binary mixtures of spherical particles, Powder Technol. 66 (1991) 259–264.
[42] A.K. Mukherjee, B.K. Mishra, An integral assessment of the role of critical process parameters on jigging, Int. J. Miner. Process. 81 (2006) 187–200, https://doi.org/10.1016/j.minpro.2006.08.005.
[43] C.R.A. Abreu, F.W. Tavares, M. Castier, Influence of particle shape on the packing and on the segregation of spherocylinders via Monte Carlo simulations, Powder Technol. 134 (2003) 167–180, https://doi.org/10.1016/S0032-5910(03)00151-7.
[44] R. Escudié, N. Epstein, J.R. Grace, H.T. Bi, Effect of particle shape on liquid-fluidized beds of binary (and ternary) solids mixtures: segregation vs. mixing, Chem. Eng. Sci. 61 (2006) 1528–1539, https://doi.org/10.1016/j.ces.2005.08.028.
[45] H.Y. Sohn, C. Moreland, The Effect of Particle Size Distribution on Packing Density, Can. J. Chem. Eng. 46 (1968) 162–167.
[46] B. Zhao, X. An, Y. Wang, Q. Qian, X. Yang, X. Sun, DEM dynamic simulation of tetrahedral particle packing under 3D mechanical vibration, Powder Technol. 317 (2017) 171–180, https://doi.org/10.1016/j.powtec.2017.04.048.